COLLECTIVE PROFITABILITY IN SEMI-COMPETITIVE INTERMEDIATION NETWORKS

Amelia Bădică

University of Craiova
Department of Business Informatics

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

CONTEXT

- Economics and Computer Science research was interested in modeling of *intermediation networks* both *technically* and *conceptually*.

- Economists: *Decentralized trading markets*: "*tradition in economics sees markets as populated by agents interacting anonymously through the price system*" (*Galeotti & Condorelli, 2016*)

- Computer Science: *Digital environments* where agents can register capabilities and search partners – implementation of decentralized trading markets.

INTERMEDIATION

- Connecting requesters with providers is known as the *connection problem* in MAS. It requires the use of specialized *middle-agents*. Their activity is called *intermediation* (*Decker et al.,1997; Bădică 2011*).

- "*Trade in a wide range of markets involves a plethora of other subjects, such as intermediaries, dealers, brokers, market-makers, wholesalers, retailers*" (*Spulber 1999*). They are sometimes called "*middlemen*".

ROLE OF INTERMEDIATION

- "*Long intermediation chains play a vital role in the market for agricultural goods in developing countries, as well as in financial markets*"

- "*Complex processes of production and distribution lead to supply chains, a natural example of chains of intermediaries*" (*Galeotti & Condorelli, 2016*)

RESEARCH QUESTIONS

- Most existing research is set in competitive context:
  - – Network formation ?
  - – Network impact on trading outcomes ?
  - – Intermediation power in resale networks ?

- Approaches:
  - – Complex networks
  - – Bargaining games

DISTRIBUTION CHAINS

- Businesses (e.g. manufacturer, wholesaler) use *intermediation networks* for the management of their distribution.
- They define *multiple distribution channels*, sometimes working simultaneously.
- A *distribution channel* contains a set of one or more market intermediaries.
- *Market intermediary* = agent that links a seller to a customer or to another intermediary with the overall goal of linking sellers with buyers.

SEMI-COMPETITIVE INTERMEDIATION

- Participant agents:
  - are collaborating to ensure that the underlying business process is achieving its functions.
  - are self-interested seeking to maximize their own profit and improve their longer-term welfare

- Goal: define correctness criteria that guarantee *collective profitability* as participants' incentive to engage in collaborative intermediation, ensuring robustness and sustainability.

RELEVANCE

- To serve both single & multiple company (consortia) distribution processes.

- B2B models of *industrial consortia* and *private industrial networks* that promote network-based B2B e-commerce as a form of “extended enterprises” (*Laudon & Traver, 2010*).

REQUIREMENTS

1. A seller can use *multiple different distribution channels*, or equivalently multiple different distribution channels can serve the same seller. This enables sellers to expand their horizon.
2. For efficiency reasons, *distribution channels can share market intermediaries* or equivalently a market intermediary can serve multiple distribution channels.
3. A business can use *multiple seller agents* to better reach the market for both reasons of efficiency and market horizon expansion.

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

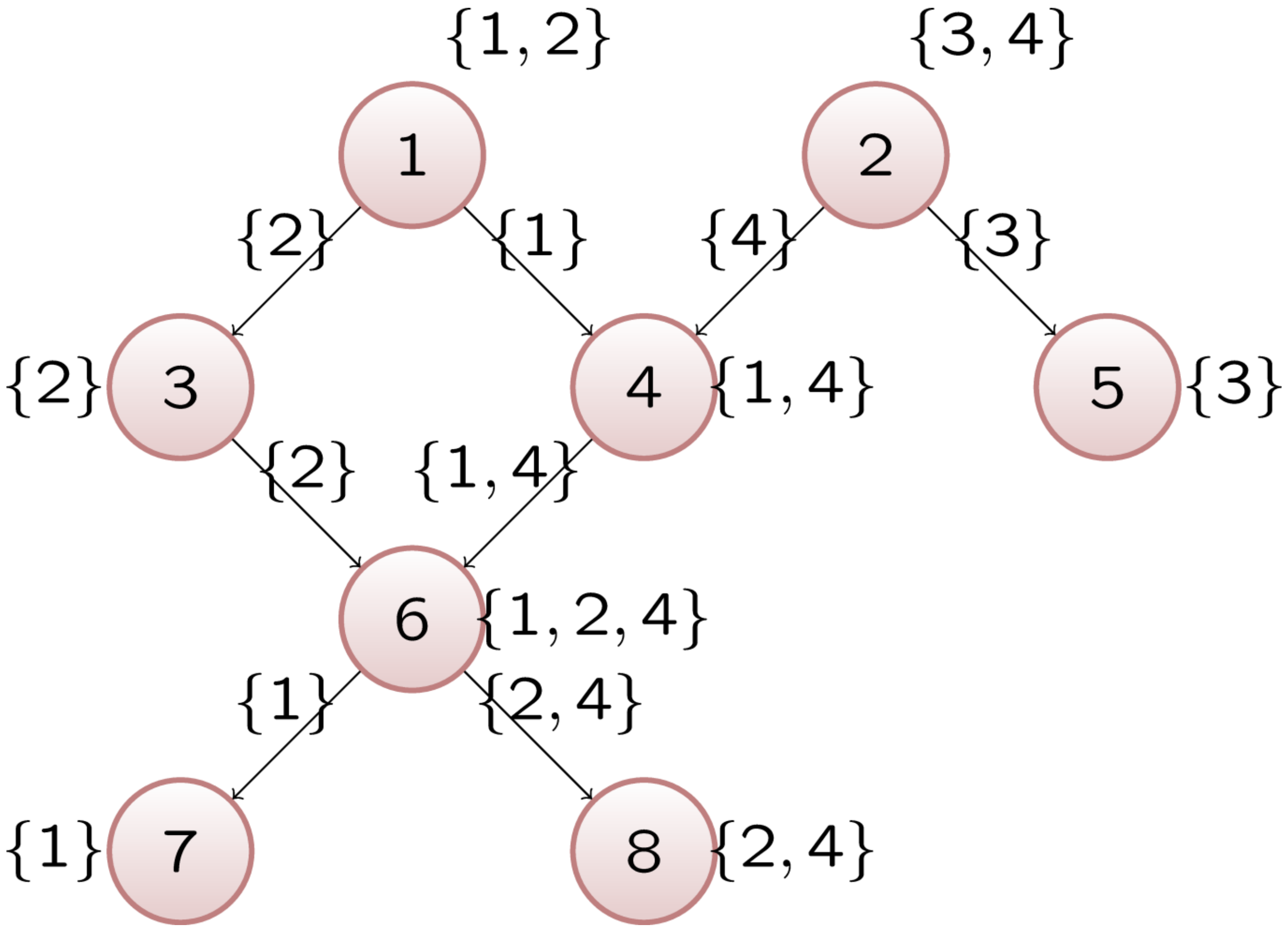
General Setting – Agents & Products

- A business brings a nonempty *set of products or services* $\mathcal{P}$ to the market.
- The business uses a *set of seller agents* $\mathcal{S}$ s.t. each seller $s \in \mathcal{S}$ distributes a nonempty set of products $P_s \subseteq \mathcal{P}$.There are no overlapping duties, so $\{P_s\}_{s \in \mathcal{S}}$ is a partition of $\mathcal{P}$.
- $\mathcal{B}$ is *set of buyers* aiming to purchase products from $\mathcal{S}$ s.t. each buyer $b \in \mathcal{B}$ wants to purchase a nonempty set $P_b \subseteq \mathcal{P}$. Purchases do not overlap so $\{P_b\}_{b \in \mathcal{B}}$ is a partition of $\mathcal{P}$.
- The business is using a *set of intermediaries* $\mathcal{I}$ that connect sellers with buyers. Each intermediary $i \in \mathcal{I}$ has dual role of buyer and seller.

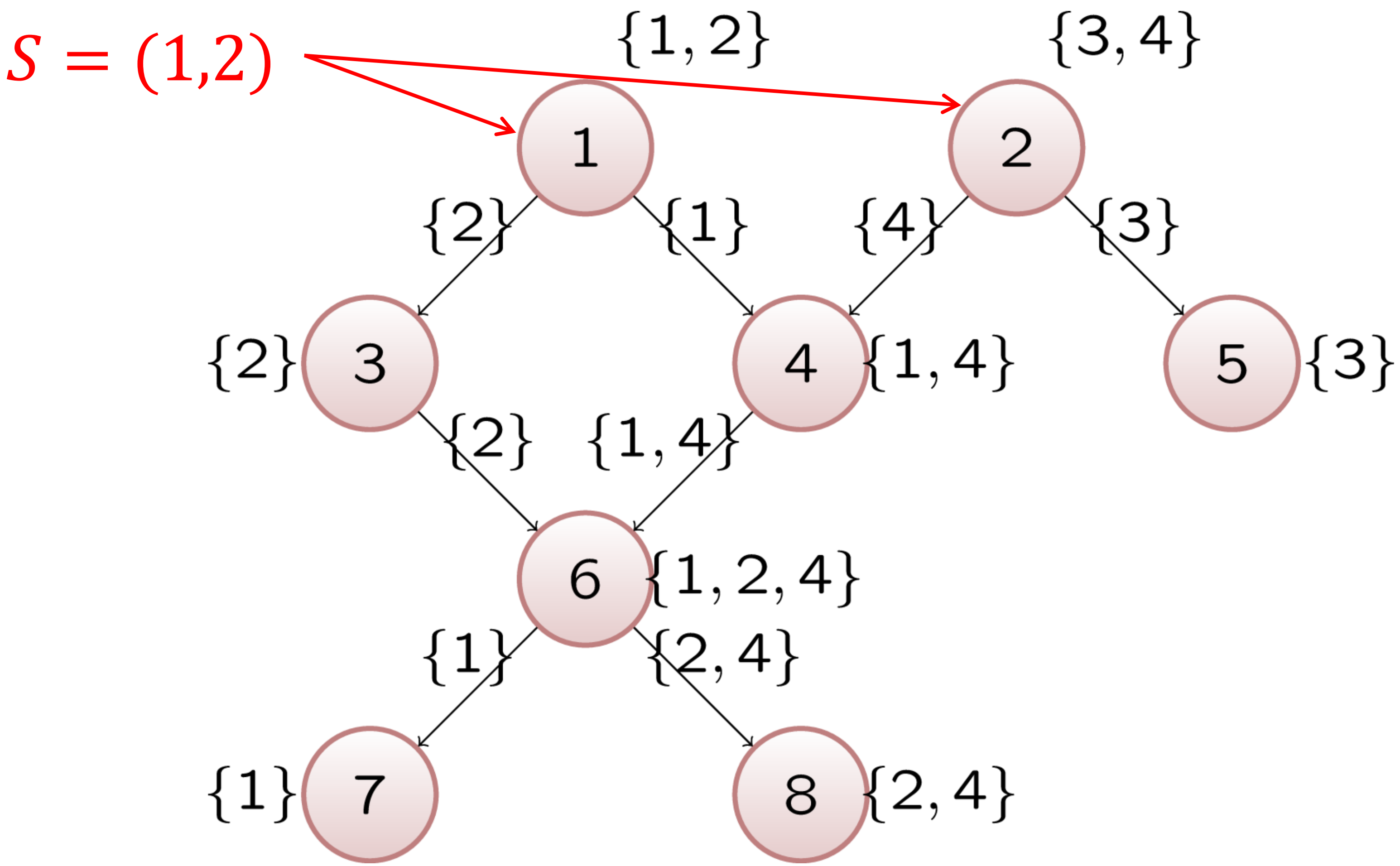
INTERMEDIATION DAG

- $\mathcal{S}, \mathcal{B},$ and $\mathcal{I}$ are finite, nonempty, pairwise disjoint sets of sellers, buyers, intermediaries. $\mathcal{P}$ is finite set of products.
- *Intermediation DAG* is quadruple $G = \langle \mathcal{V}, \mathcal{A}, p, g \rangle$ such that:
- $\langle \mathcal{V}, \mathcal{A} \rangle$ is a DAG with the set of vertices $\mathcal{V} = \mathcal{S} \cup \mathcal{B} \cup \mathcal{I}$ s.t.:
  - $in(s) = \emptyset$ for all $s \in \mathcal{S}$ and $out(b) = \emptyset$ for all $b \in \mathcal{B}$
  - $in(i) \neq \emptyset$ and $out(i) \neq \emptyset$ for all $i \in \mathcal{I}$
- $p: \mathcal{V} \rightarrow 2^{\mathcal{P}} \setminus (\emptyset)$ and $g: \mathcal{A} \rightarrow 2^{\mathcal{P}} \setminus (\emptyset)$ are two functions mapping each agent (node) and each transaction (arc) to a nonempty set of products such that:
  - $(g((u,v)))_{v \in out(u)}$ is a nontrivial partition of set $p(u)$ for all $u \in \mathcal{S} \cup \mathcal{I}$
  - $(g((v,u)))_{v \in in(u)}$ is a nontrivial partition of set $p(u)$ for all $u \in \mathcal{B} \cup \mathcal{I}$
  - $\cup_{s \in \mathcal{S}} p(s) = \cup_{b \in \mathcal{B}} p(b) = \mathcal{P}$

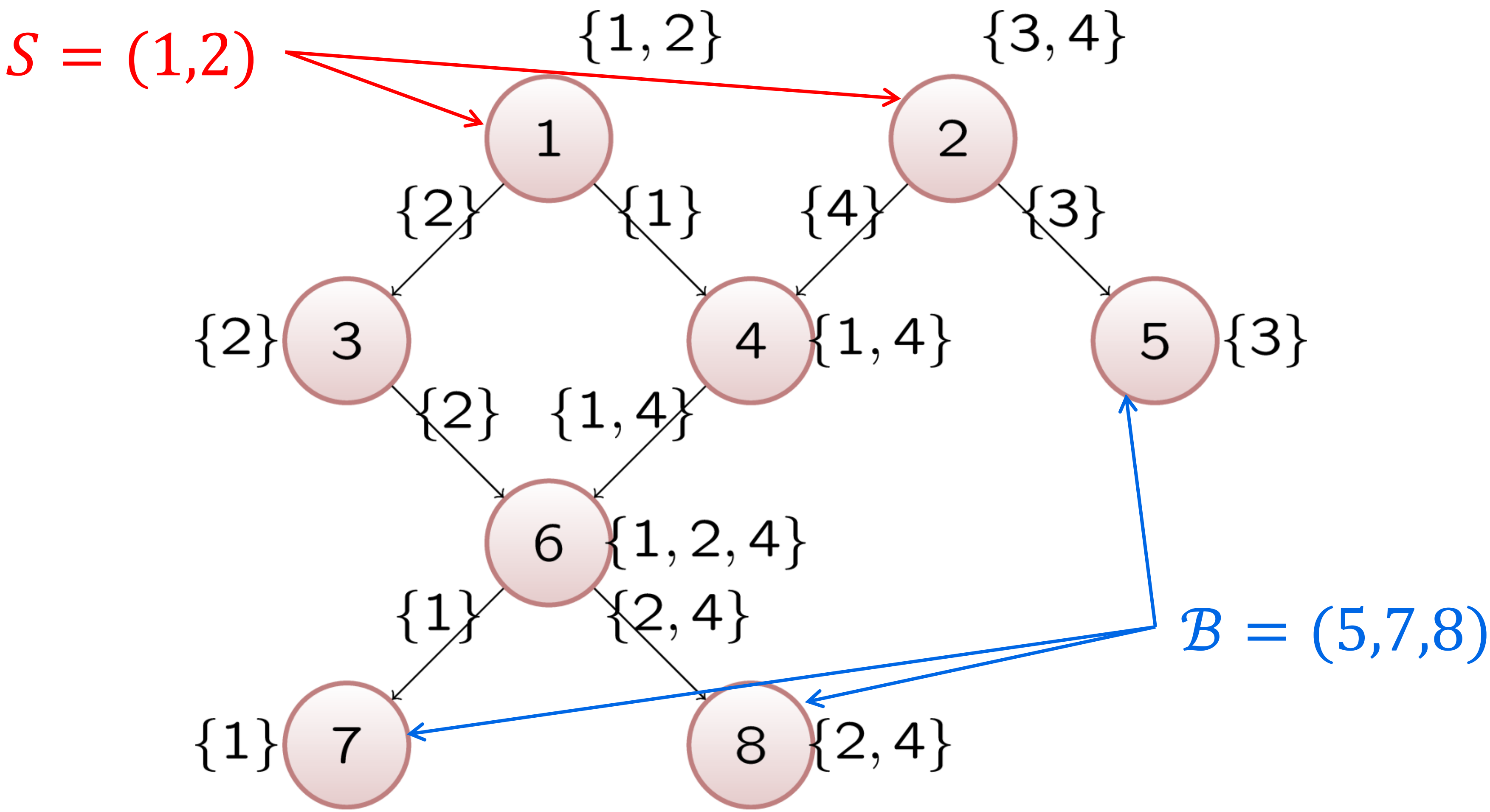
Sample Intermediation Graph



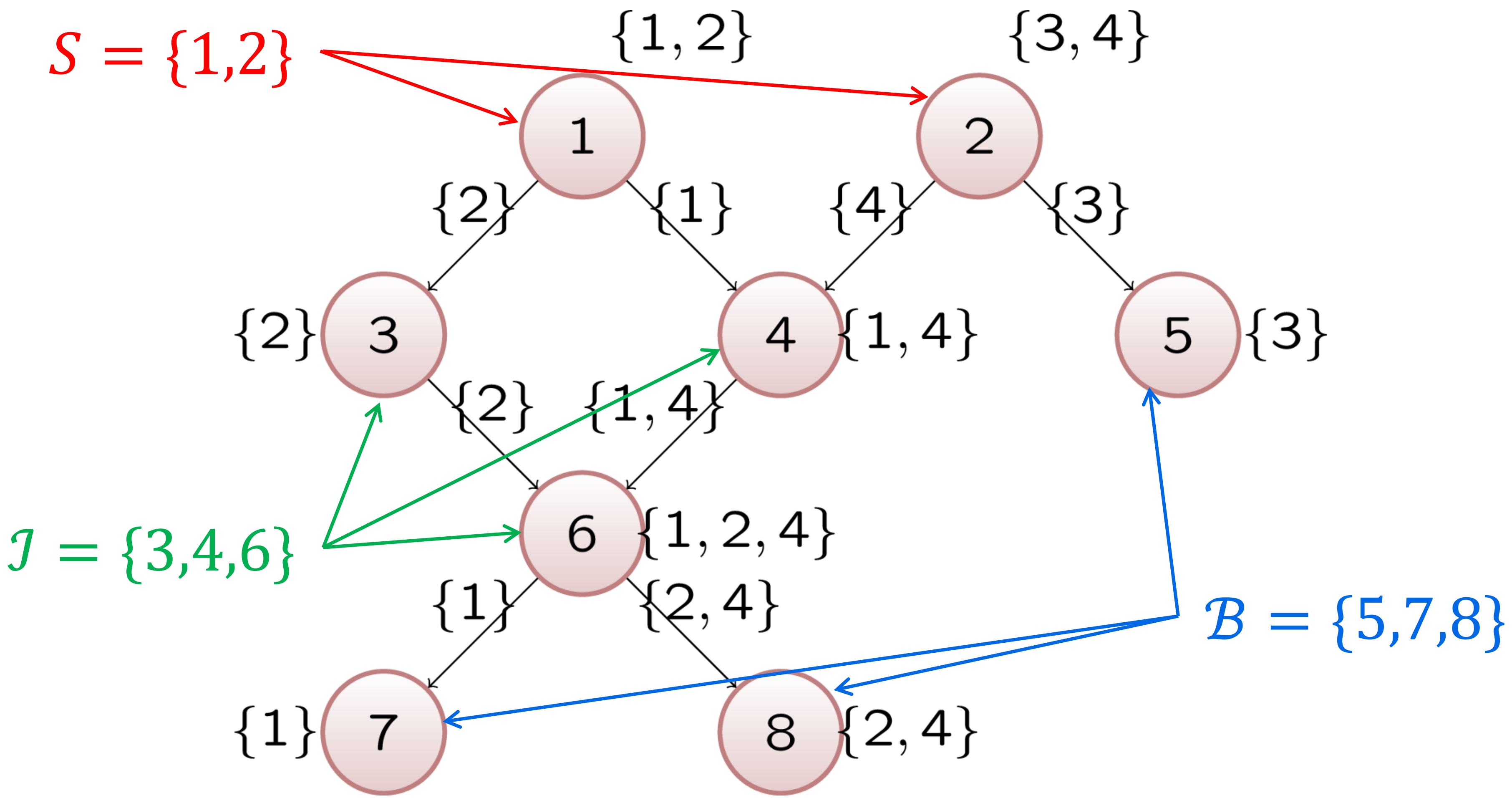
SAMPLE INTERMEDIATION GRAPH



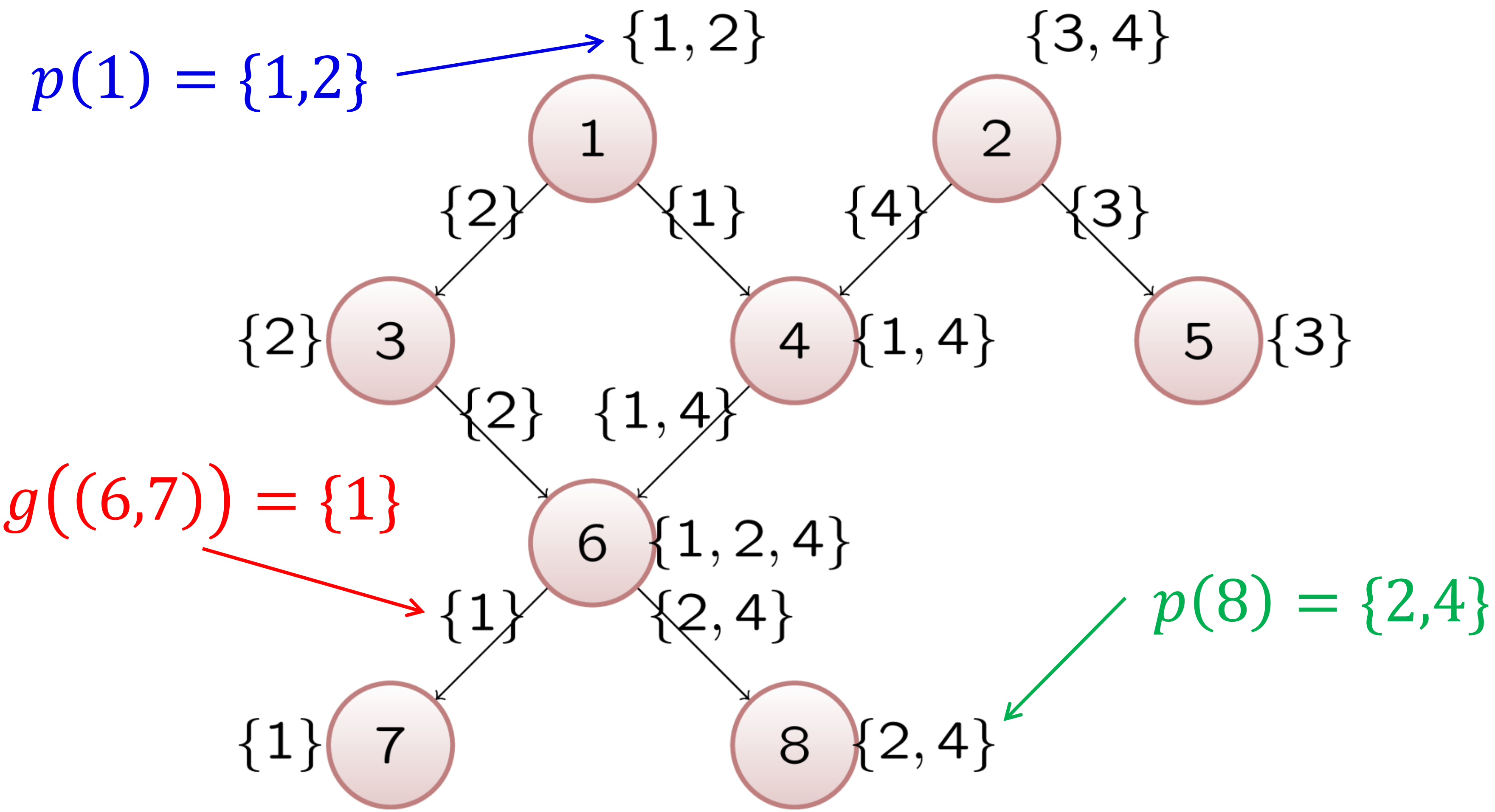
SAMPLE INTERMEDIATION GRAPH



SAMPLE INTERMEDIATION GRAPH



SAMPLE INTERMEDIATION GRAPH



Number of Transactions

- **Proposition 1**. The total number of transactions t of a weakly connected intermediation DAG satisfies the inequality:

$$t \geq |\mathcal{S}| + |\mathcal{B}| + |\mathcal{I}| - 1$$

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

LIMIT PRICE

- σ_P^s is the *limit price of seller* s for selling set P of products. s agrees to sell P only for a price x s.t.:

$$x \geq \sigma_P^s$$

- β_P^b is the *limit price of a buyer* b agreeing to pay and buy set P of products. b agrees to purchase P only for a price x s.t.:

$$x \leq \beta_P^b$$

ANNOTATED INTERMEDIATION DAG

- Given intermediation DAG $G = \langle \mathcal{V}, \mathcal{A}, p, g \rangle$, the annotated version of G is $G^a = \langle \mathcal{V}, \mathcal{A}, p, g, \pi \rangle$ s.t. $\pi: \mathcal{S} \cup \mathcal{B} \cup \mathcal{A} \rightarrow (0, +\infty)$ is the *annotation function with economic information* defined as:
  - If $s \in \mathcal{S}$ is a seller then $\pi(s) = \sigma_{p(s)}^{s} > 0$ is the limit price of seller s for selling products $p(s)$
  - If $b \in \mathcal{B}$ is a buyer then $\pi(b) = \beta_{p(b)}^{b} > 0$ is the limit price of buyer b for purchasing products $p(b)$
  - If $t = (u, v) \in \mathcal{A}$ denotes a transaction then $\pi(t) = \pi_{g((u,v))}^{u,v} > 0$ is the transaction price for which agent u agrees to sell products $g((u, v))$ to agent v.

GENERAL RULE: MUTUAL GAIN

- Assume seller s with limit price σ transacts with buyer b with limit price β.
- If s transacts at price π then its utility is $\pi - \sigma \geq 0$ so:

$$\pi \geq \sigma$$

- If b transacts at price π then its utility is $\beta - \pi \geq 0$ so:

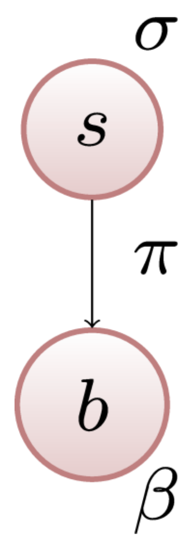
$$\beta \geq \pi$$

- Combining inequalities, the transaction holds if and only if:

$$\beta \geq \sigma$$

 at any price $\pi \in [\sigma, \beta]$.

Simplest nontrivial intermediation DAG



PARTICIPANTS' UTILITIES

- If $s \in \mathcal{S}$ is seller agent then its utility is:

$$u(s) = -\sigma_{p(s)}^{s} + \sum_{v \in out(s)} \pi_{g((s,v))}^{s,v}$$

- If $b \in \mathcal{B}$ is buyer agent then its utility is:

$$u(b) = \beta_{p(b)}^{b} - \sum_{v \in in(b)} \pi_{g((v,b))}^{v,b}$$

- If $i \in \mathcal{I}$ is intermediary agent then its utility is:

$$u(i) = \sum_{v \in out(i)} \pi_{g((i,v))}^{i,v} - \sum_{v \in in(i)} \pi_{g((v,i))}^{v,i}$$

COLLECTIVE PROFITABILITY

- **Definition.** An intermediation DAG is called *collectively profitable* iff it can be annotated with transaction prices s.t. each participant v is profitable, i.e. it gains or at least it does not lose by performing the transaction:

$$u(v) \geq 0$$

COLLECTIVE PROFITABILITY CONDITION

- **Lemma.** An intermediation DAG is *collectively profitable* iff there exist transaction prices satisfying the system of $|\mathcal{V}| = |\mathcal{S}| + |\mathcal{B}| + |\mathcal{I}|$ inequalities and $t = |\mathcal{A}|$ unknowns:

$$-\sigma_{p(s)}^{s} + \sum_{v \in out(s)} \pi_{g((s,v))}^{s,v} \geq 0, \qquad s \in \mathcal{S}$$

$$\beta_{p(b)}^{b} - \sum_{v \in in(b)} \pi_{g((v,b))}^{v,b} \geq 0, \qquad b \in \mathcal{B}$$

$$\sum_{v \in out(i)} \pi_{g((i,v))}^{i,v} - \sum_{v \in in(i)} \pi_{g((v,i))}^{v,i} \geq 0, \qquad i \in \mathcal{I}$$

EXAMPLE

$$-\sigma_{1,2}^{1} + \pi_2^{1,3} + \pi_1^{1,4} \geq 0$$

$$-\sigma_{3,4}^{2} + \pi_2^{2,4} + \pi_3^{2,5} \geq 0$$

$$-\pi_2^{1,3} + \pi_2^{3,6} \geq 0$$

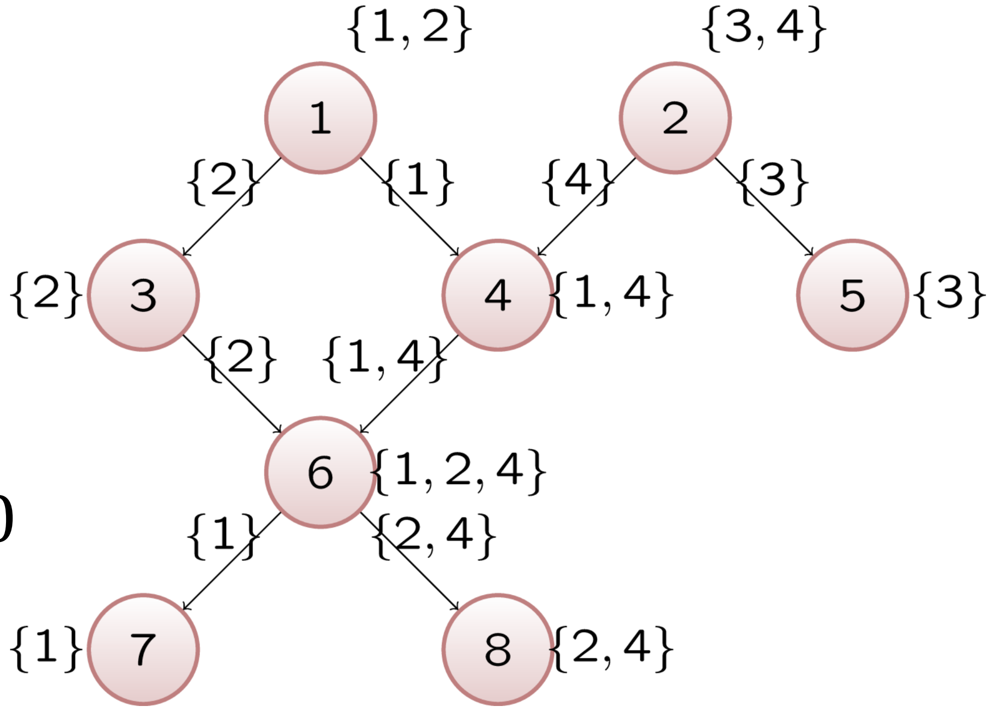
$$-\pi_1^{1,4} - \pi_4^{2,4} + \pi_{1,4}^{4,6} \geq 0$$

$$\beta_3^5 - \pi_3^{2,5} \geq 0$$

$$-\pi_2^{3,6} - \pi_{1,4}^{4,6} + \pi_1^{6,7} + \pi_{2,4}^{6,8} \geq 0$$

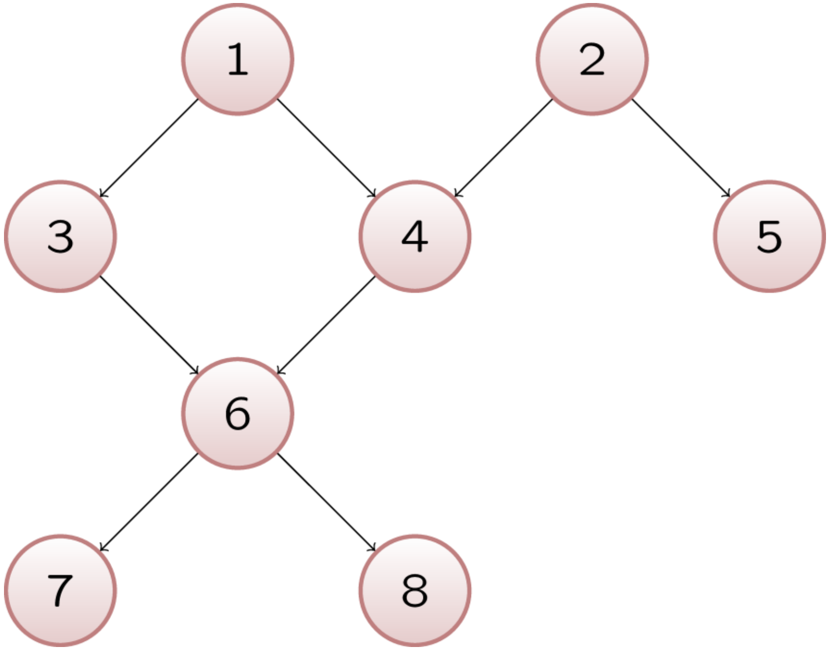
$$\beta_1^7 - \pi_1^{6,7} \geq 0$$

$$\beta_{2,4}^8 - \pi_{2,4}^{6,8} \geq 0$$



Example – Products Omitted

$$-\sigma_1 + \pi_{13} + \pi_{14} \geq 0$$
$$-\sigma_2 + \pi_{24} + \pi_{25} \geq 0$$
$$-\pi_{13} + \pi_{36} \geq 0$$
$$-\pi_{14} - \pi_{24} + \pi_{46} \geq 0$$
$$\beta_5 - \pi_{25} \geq 0$$
$$-\pi_{36} - \pi_{46} + \pi_{67} + \pi_{68} \geq 0$$
$$\beta_7 - \pi_{67} \geq 0$$
$$\beta_8 - \pi_{68} \geq 0$$



TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

LOOK FOR "USABLE" CONDITIONS

- More "usable" conditions for collective profitability?
- <u>Our results:</u>
  1. Collective profitability "reduces" to checking a set of inequalities involving buyer and seller limit prices.
  2. This set depends on the DAG structure.
- Defining this set assumes the steps:
  1. Introduce helper functions $reachable$ and $leaves$.
  2. Define "matching pairs" $(buyers, sellers)$
  3. Generate a condition (inequality) for each matching pair.

Reachable and Leaf Nodes of a DAG

- Function $reachable : 2^{\mathcal{V}} \to 2^{\mathcal{V}}$ maps each set of nodes $W \subseteq \mathcal{V}$ to the set of nodes reachable from W.

  $$reachable(\emptyset) = \emptyset$$

  $$reachable(W) = W \cup \bigcup_{w \in W} reachable(out(w)))$$

- Function $leaves : 2^{\mathcal{V}} \to 2^{\mathcal{V}}$ maps each set of nodes $W \subseteq \mathcal{V}$ to set of leaf nodes reachable from W.

  $$leaves(W) = reachable(W) \cap \mathcal{B}$$

MATCHING PAIRS

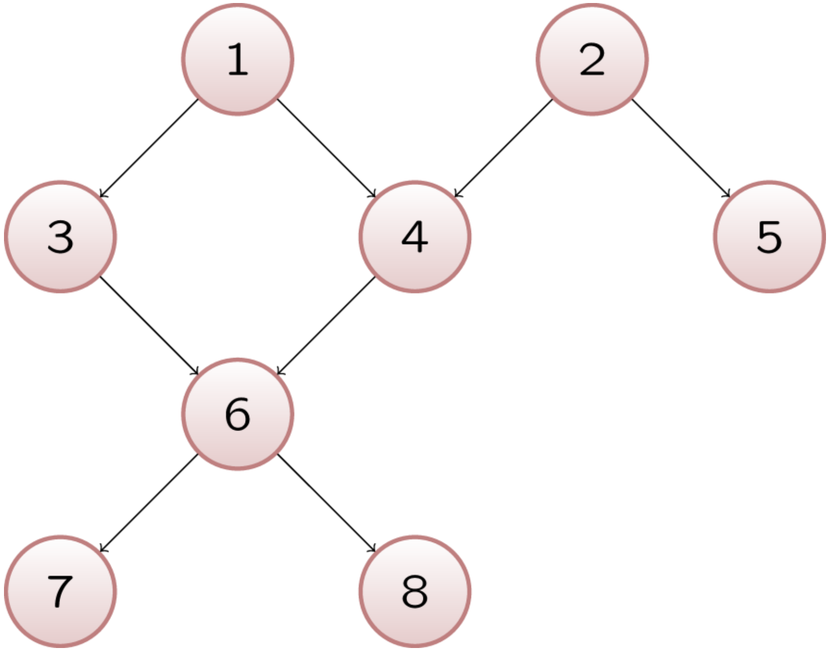
- *Matching pair*: (B, S_B) such that:
  - $B \subseteq \mathcal{B}$ and $S_B \subseteq \mathcal{S}$
  - $leaves(S_B) = B$
  - S_B is maximal with respect to set inclusion (*a not maximal* S_B *generates a redundant inequality*)
- Let $\mathbb{P}$ be the *set of matching pairs* of an intermediation DAG.

EXAMPLE

$$reachable(\{2\}) = \{2, 4, 5, 6, 7, 8\}$$

$$leaves(\{2\}) = leaves(\{1, 2\}) = \{5, 7, 8\}$$

$$\mathbb{P} = \{(\{7, 8\}, \{1\}), (\{5, 7, 8\}, \{1, 2\})\}$$



NECESSARY CONDITION

- **Proposition.** Let us consider an intermediation DAG $G = \langle \mathcal{V}, \mathcal{A}, p, g \rangle$ and let $\mathbb{P}_G$ be its set of matching pairs. If G is collectively profitable then the following linear homogenous inequations hold:

$$\forall \, (B, S_B) \in \mathbb{P}_G \qquad \sum_{b \in B} \beta^b_{p(b)} \geq \sum_{s \in S_B} \sigma^s_{p(s)}$$

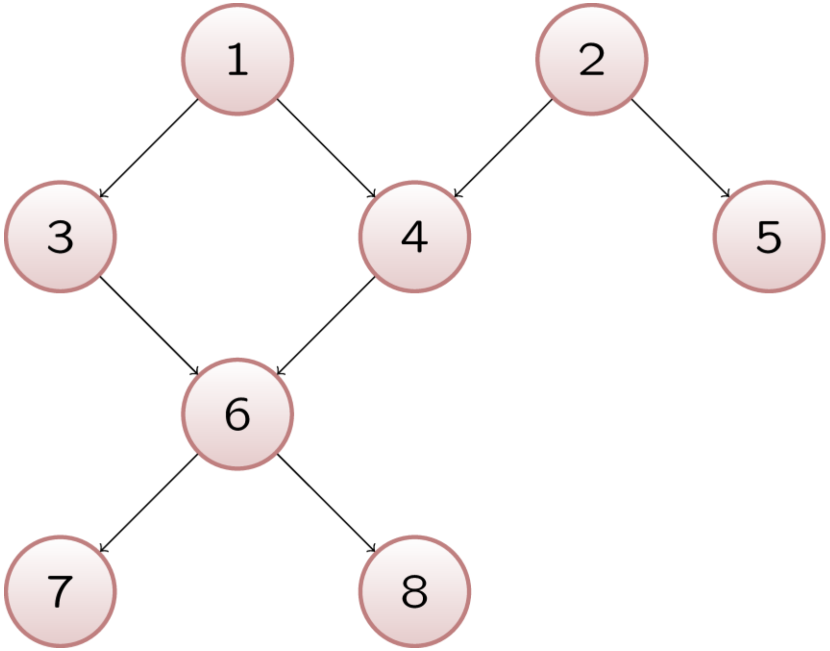
- **Observation.** For a single-rooted (s is the root) weakly connected intermediation DAG (includes trees !):

$$\sum_{b \in \mathcal{B}} \beta^b_{p(b)} \geq \sigma^s_{p(s)}$$

EXAMPLE

$$\beta_1^7 + \beta_{2,4}^8 + \beta_3^5 \geq \sigma_{1,2}^1 + \sigma_{3,4}^2 \qquad \beta_7 + \beta_8 + \beta_5 \geq \sigma_1 + \sigma_2$$

$$\beta_1^7 + \beta_{2,4}^8 \geq \sigma_{1,2}^1 \qquad \beta_7 + \beta_8 \geq \sigma_1$$



SUFFICIENT CONDITION

- **Proposition.** If a single rooted intermediation DAG $G = \langle \mathcal{V}, \mathcal{A}, p, g \rangle$ with root s satisfies:

  $$\sum_{b \in \mathcal{B}} \beta_{p(b)}^{b} \geq \sigma_{p(s)}^{s}$$

  then it is collectively profitable.

- Mathematical proof is achieved using Farkas lemma (1902) that states conditions when a system of linear equations has positive solutions.

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
  - Computational Limitations
  - Case Study
- Conclusions and Future Works

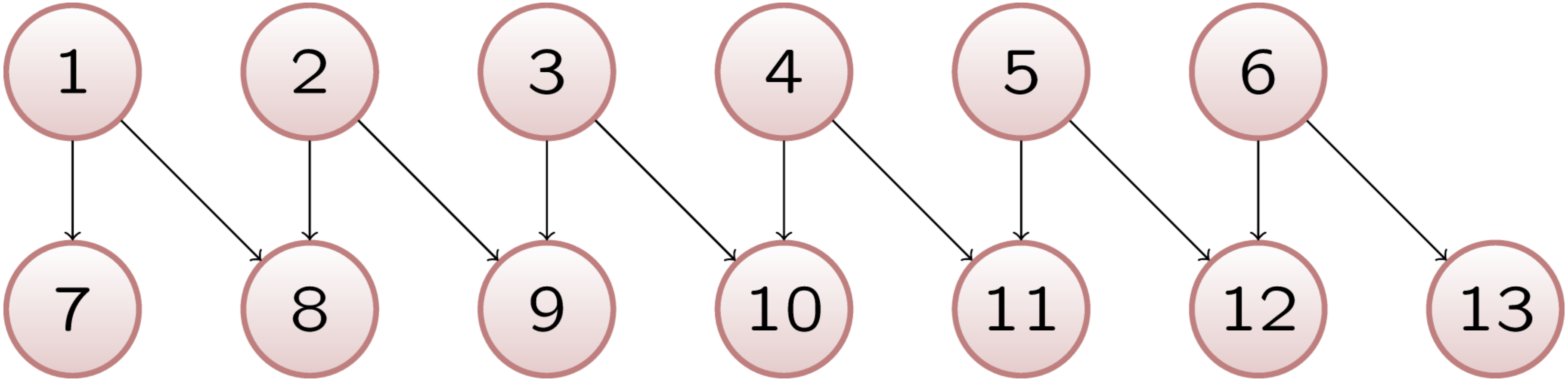
COMPUTATIONAL LIMITATIONS

- What about the applicability of the collective profitability conditions.

What are the theoretical and practical limits of using them ?

Many Matching Pairs

- $\mathcal{V} = \{1,2, \dots, 2N+1\}, N = 4n-2, n \geq 1$
- $\mathcal{S} = \{1,2, \dots, N\}, \mathcal{B} = \{1,2, \dots, N+1\}$
- $\mathcal{A} = \{(i, i+N), (i, i+N+1) \mid i = 1,2, \dots, N\}$



$$n = 2, \qquad N = 6$$

MATCHING PAIRS

- Let us consider subsets of sellers $S = \{i_1, i_2, \dots\ i_n\}$ defined for $i_j \in \{4j-3, 4j\ -2\}$ for all $j = 1, 2, \dots, n$.
- Then:

$$leaves(S) =$$

$$\{i_1 + N, i_1 + N + 1, i_2 + N, i_2 + N + 1, \dots ,$$

$$i_n + N, i_n + N + 1\}$$

- *Each pair $(leaves(S), S)$ is a matching pair.*

EXAMPLE

- The subsets are $\{i_1, i_2\}$ with $i_1 \in \{1,2\}$ and $i_2 \in \{5,6\}$:

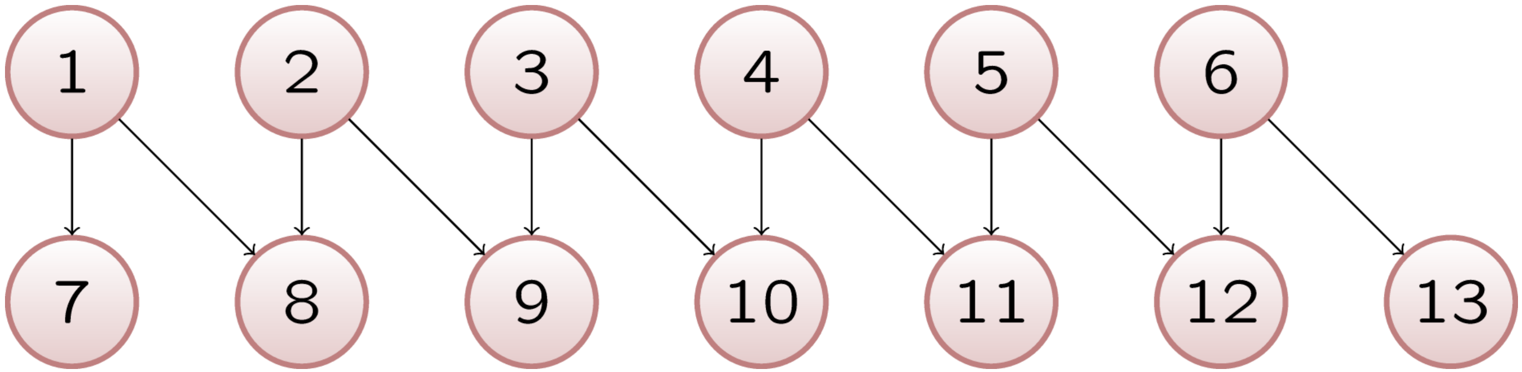
  $\{1,5\}$, $\{2,5\}$, $\{1,6\}$, $\{2,6\}$
- Matching pairs are:

  $(\{7,8,11,12\}, \{1,5\})$

  $(\{8,9,11,12\}, \{2,5\})$

  $(\{7,8,12,13\}, \{1,6\})$

  $(\{8,9,12,13\}, \{2,6\})$



INTRACTABILITY

- We have defined 2^n matching pairs for the example graph with $N = 4n - 2$ and $k = 2N + 1 = 8n - 5$ nodes.

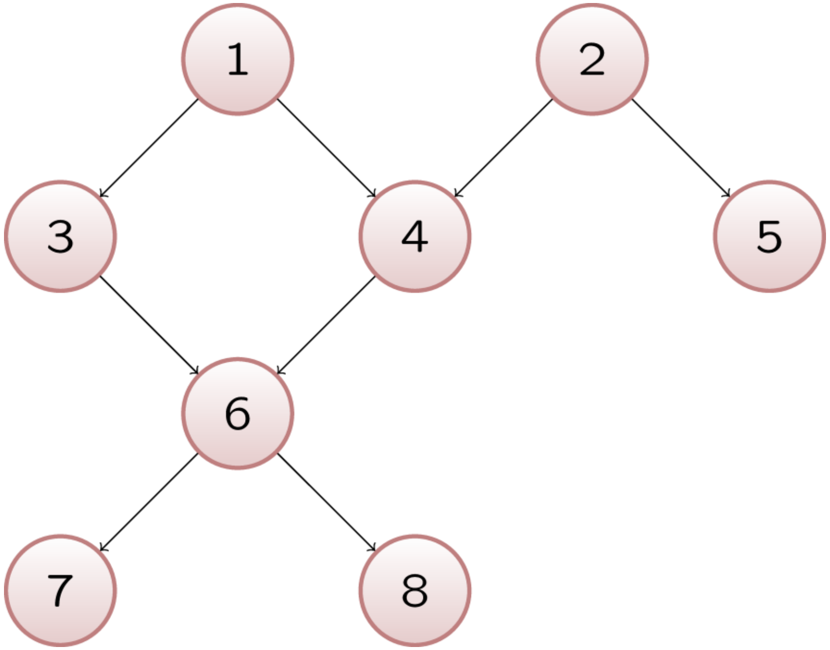
- So, for a graph with k nodes we found <u>at least</u> $2^{\frac{k}{8}}$ matching pairs $\Rightarrow$ *the number of inequalities can be exponential in the number of nodes for some intermediation graphs !*

CASE STUDY

If these inequalities hold then the intermediation DAG is collectively profitable.

$$\beta_7 + \beta_8 + \beta_5 \geq \sigma_1 + \sigma_2$$

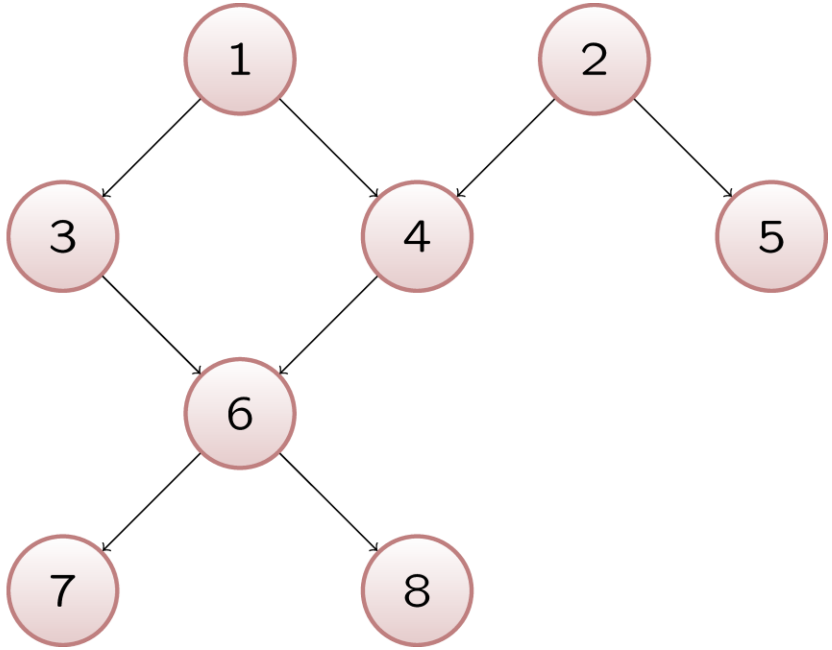
$$\beta_7 + \beta_8 \geq \sigma_1$$



Solve Inequalities

- Show that if conditions hold then system of inequalities has positive solutions

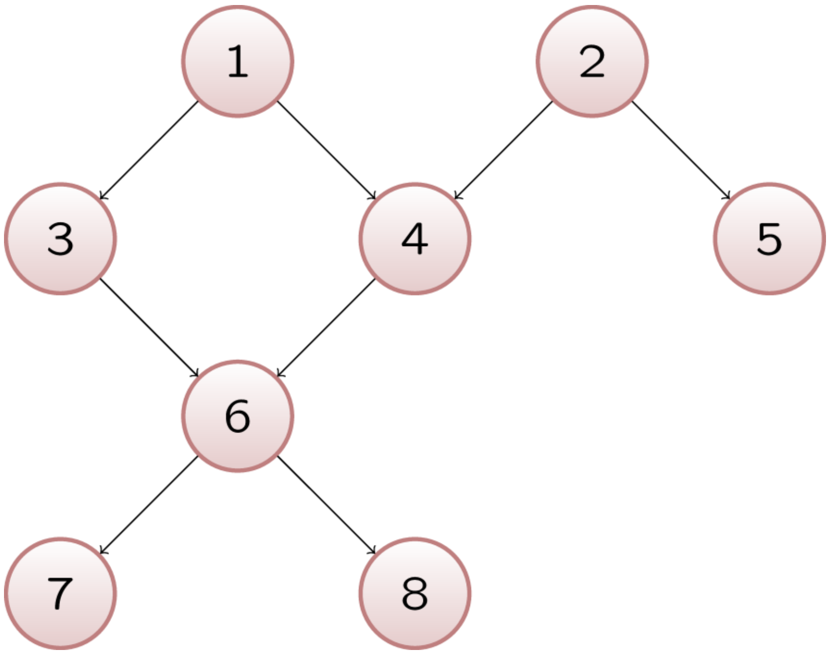
$$-\sigma_1 + \pi_{13} + \pi_{14} \geq 0$$
$$-\sigma_2 + \pi_{24} + \pi_{25} \geq 0$$
$$-\pi_{13} + \pi_{36} \geq 0$$
$$-\pi_{14} - \pi_{24} + \pi_{46} \geq 0$$
$$\beta_5 - \pi_{25} \geq 0$$
$$-\pi_{36} - \pi_{46} + \pi_{67} + \pi_{68} \geq 0$$
$$\beta_7 - \pi_{67} \geq 0$$
$$\beta_8 - \pi_{68} \geq 0$$



Reduce to Equations

- There exists $\alpha_i \geq 0$ such that system of equalities has positive solutions.

$$-\sigma_1 + \pi_{13} + \pi_{14} = \alpha_1$$
$$-\sigma_2 + \pi_{24} + \pi_{25} = \alpha_2$$
$$-\pi_{13} + \pi_{36} = \alpha_3$$
$$-\pi_{14} - \pi_{24} + \pi_{46} = \alpha_4$$
$$\beta_5 - \pi_{25} = \alpha_5$$
$$-\pi_{36} - \pi_{46} + \pi_{67} + \pi_{68} = \alpha_6$$
$$\beta_7 - \pi_{67} = \alpha_7$$
$$\beta_8 - \pi_{68} = \alpha_8$$



8 equations, but only 7 are independent.
Solve it taking one unknown as parameter: π_{14}

Solve System Of Equations

- Letting:

$\Delta_1 = \beta_7 + \beta_8 - \sigma_1 \geq 0$

$\Delta_2 = \beta_7 + \beta_8 + \beta_5 - \sigma_1 - \sigma_2 \geq 0$ Note that: $\Delta_2 = \sum_{i=1}^{8} \alpha_i$

we obtain:

$\pi_{13} = \alpha_1 + \sigma_1 - \pi_{14}$

$\pi_{14} = \pi_{14}$

$\pi_{24} = \Delta_1 - \alpha_1 - \alpha_3 - \alpha_4 - \alpha_6 - \alpha_7 - \alpha_8$

$\pi_{25} = \beta_5 - \alpha_5$

$\pi_{36} = \sigma_1 + \alpha_1 + \alpha_3 - \pi_{14}$

$\pi_{46} = \Delta_1 - \alpha_1 - \alpha_3 - \alpha_6 - \alpha_7 - \alpha_8 + \pi_{14}$

$\pi_{67} = \beta_7 - \alpha_7$

$\pi_{68} = \beta_8 - \alpha_8$

POSITIVITY CONSTRAINTS

$$\alpha_1 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \alpha_8 < \Delta_1$$
$$\alpha_8 < \beta_8$$
$$\alpha_7 < \beta_7$$
$$\alpha_5 < \beta_5$$
$$\pi_{14} < \sigma_1$$
$$\alpha_1 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 < \Delta_2$$

- We can choose α_i and π_{14} s.t. :

$$\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = h > 0$$
$$\alpha_2 = \Delta_2 - 7h$$
$$\pi_{14} = p > 0$$
$$0 < h < \min\{\beta_5, \beta_7, \beta_8, \frac{\Delta_1}{6}, \frac{\Delta_2}{7}\}$$
$$0 < p < \sigma_1$$

Solutions

- Here they are:

$$\pi_{13} = h + \sigma_1 - p$$
$$\pi_{14} = p$$
$$\pi_{24} = \Delta_1 - 6h$$
$$\pi_{25} = \beta_5 - h$$
$$\pi_{36} = \sigma_1 + 2h - p$$
$$\pi_{46} = \Delta_1 - 5h + p$$
$$\pi_{67} = \beta_7 - h$$
$$\pi_{68} = \beta_8 - h$$

- Remember conditions:

$$\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = h$$
$$\alpha_2 = \Delta_2 - 7h$$
$$\pi_{14} = p$$
$$0 < h < \min\{\beta_5, \beta_7, \beta_8, \frac{\Delta_2}{6}, \frac{\Delta_1}{7}\}$$
$$0 < p < \sigma_1$$

TALK OVERVIEW

- Introduction and Motivation
- Network-Based Model of Intermediation
- Collectively Profitable Networks
- Collective Profitability Conditions
- Discussion
- Conclusions and Future Works

CONCLUSIONS

1. Formal model of intermediation that is able to serve multiple distribution channels working simultaneously and possibly sharing intermediary agents as an *intermediation DAG*.
2. *Necessary and sufficient conditions for collective profitability* of an intermediation DAG, as systems of linear inequalities involving limit prices of buyer and seller agents.
3. An example showing that *the number of inequality conditions can grow exponentially with the number of agents* in the intermediation DAG.

FUTURE WORKS

1. Apply the concepts of welfare economics to analyze *optimal pricing strategies of the transaction participants*.
2. Propose practical *computational methods to determine optimal pricing strategies*.
3. Study the *stability of pricing strategies* using the concepts of game theory.

Thank You